Resonant Interactions Between Protons and Oblique Alfvén/Ion-Cyclotron Waves

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Abstract. Resonant interactions between ions and Alfvén/ion-cyclotron (A/IC) waves may play an important role in the heating and acceleration of the fast solar wind. Although such interactions have been studied extensively for "parallel" waves, whose wave vectors $\mathbf{k}$ are aligned with the background magnetic field $\mathbf{B}_0$, much less is known about interactions between ions and oblique A/IC waves, for which the angle θ between $\mathbf{k}$ and $\mathbf{B}_0$ is nonzero. In this paper, we present new numerical results on resonant cyclotron interactions between protons and oblique A/IC waves in collisionless low-beta plasmas such as the solar corona. We find that if some mechanism generates oblique high-frequency A/IC waves, then these waves initially modify the proton distribution function in such a way that it becomes unstable to parallel waves. Parallel waves are then amplified to the point that they dominate the wave energy at the large parallel wave numbers at which the waves resonate with the particles. Pitch-angle scattering by these waves then causes the plasma to evolve towards a state in which the proton distribution is constant along a particular set of nested "scattering surfaces" in velocity space, whose shapes have been calculated previously. As the distribution function approaches this state, the imaginary part of the frequency of parallel A/IC waves drops continuously towards zero, but oblique waves continue to undergo cyclotron damping while simultaneously causing protons to diffuse across these kinetic shells to higher energies. We conclude that oblique A/IC waves can be more effective at heating protons than parallel A/IC waves, because for oblique waves the plasma does not relax towards a state in which proton damping of oblique A/IC waves ceases.

Keywords: solar wind, solar corona, quasilinear theory, wave-particle interaction, plasma turbulence, ion cyclotron waves, ion heating
PACS: 94.05.Pt, 96.25.Qr, 96.50.Ci, 96.60.P-

INTRODUCTION

Resonant interactions with Alfvén/ion-cyclotron (A/IC) waves are a possible mechanism for ion heating in the solar corona, solar flares, and the solar wind. Cyclotron heating in low-β plasmas primarily increases a particle's thermal motions perpendicular to the background magnetic field $\mathbf{B}_0$ [1], and may thus be able to explain the observed temperature anisotropies of minor ions in the solar corona [2] and protons in the fast solar wind [3]. (Here, $\beta = 8\pi p/B_0^2$, where p is the plasma pressure.) In solar flares, the magnetic tension in reconnected magnetic field lines leads to large-scale flows that can generate waves and turbulence. Wave energy is then transfered from large scales to small scales by nonlinear wave-wave interactions [4, 5]. Small-scale A/IC waves may be sufficiently energetic in solar flares to stochastically accelerate ions to high energies [6, 7]. Most previous studies of ion heating by A/IC waves have focused on "parallel waves," for which the angle θ between the wave vector $\mathbf{k}$ and $\mathbf{B}_0$ is zero. On the other hand, in the solar corona and solar flares, A/IC wave intensities are not restricted to $\theta = 0$. In this paper, we thus focus on resonant interactions between protons and oblique A/IC waves, for which $\theta \neq 0$.

WAVE-PARTICLE INTERACTIONS

We consider A/IC waves in a low-β, proton-electron plasma, and assume that the real part of the wave frequency, ω_{kr}, is given by the cold-plasma A/IC dispersion relation [9],

$$w^2 = \frac{k_n^2}{2}(1+\cos^2\theta) + \frac{k_n^4}{2}\cos^2\theta - \frac{k_n^2}{2}\left[k_n^4\cos^4\theta + 2k_n^2\cos^2\theta(1+\cos^2\theta) + \sin^4\theta\right]^{1/2}, \tag{1}$$

where $w = \omega_{kr}/\Omega_p$, Ω_p is the proton cyclotron frequency, $k_n = kv_{\rm A}/\Omega_p$, and $v_{\rm A} = B_0/\sqrt{4\pi\rho_0}$ is the Alfvén speed. Protons strongly interact with such waves only when the resonance condition,

$$\omega_{kr} - k_\parallel v_\parallel = n\Omega_p, \tag{2}$$

is satisfied, where $v_\parallel$ ($v_\perp$) is the component of the particle velocity $\mathbf{v}$ parallel (perpendicular) to $\mathbf{B}_0$, $k_\parallel$ ($k_\perp$) is the component of $\mathbf{k}$ parallel (perpendicular) to $\mathbf{B}_0$, and n is any integer [8, 9]. The strongest interaction occurs for n=1 [1].

Figure 1 plots $w(k_n)$ for $\theta = 0$ (solid line), as well as two dashed lines corresponding to $1 + k_\parallel v_\parallel/\Omega_p$ for

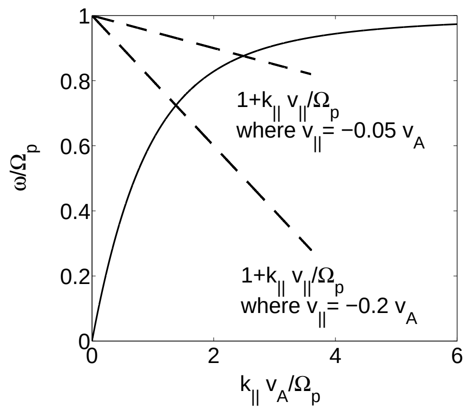

FIGURE 1. The intersections between lines satisfy the resonant condition n=1.

two different values of the proton parallel velocity $v_\parallel$. The intersections of these two lines and the $w(k_n)$ curve correspond to solutions of equation (2) with $n = 1$. When $|v_\parallel| \ll v_\mathrm{A}$, equations (1) and (2) imply that [10]

$$k_{n\parallel,\mathrm{res}} \simeq \left|\frac{v_\parallel}{v_\mathrm{A}}\right|^{-1/3}, \qquad (3)$$

and

$$v_\mathrm{ph}(v_\parallel) \simeq \pm v_\mathrm{A}^{2/3}|v_\parallel|^{1/3}, \qquad (4)$$

where $k_{n\parallel,\mathrm{res}}$ is the value of $k_\parallel v_\mathrm{A}/\Omega_p$ that satisfies equation (2) for $\theta = 0$, and $v_\mathrm{ph}(v_\parallel)$ is the parallel phase velocity $\omega/k_\parallel$ of the resonant waves at $\theta = 0$.

Resonant interactions between particles and waves cause particles to diffuse in the $v_\parallel - v_\perp$ plane. Particles interacting with a particular wave with wave vector $\mathbf{k}$ and frequency ω diffuse within the $v_\parallel - v_\perp$ plane along a curve for which the particle energy is conserved in a frame moving with velocity $\omega/k_\parallel$ in the direction of $\mathbf{B}_0$ [8, 9] (wave pitch-angle scattering). If protons interact with A/IC waves with a broad range of $k_\parallel$ values and $\theta = 0$, then particles will diffuse along closed contours in the $v_\parallel - v_\perp$ plane. These contours are defined by the equation $\eta =$ constant, where [11, 12]

$$\eta \simeq v_\perp^2 + (3/2)v_\mathrm{A}^{2/3}|v_\parallel|^{4/3}. \qquad (5)$$

If resonant wave-particle interactions control the evolution of the proton distribution function f, then f becomes constant on surfaces of constant η. Once $f = f(\eta)$, protons stop gaining or losing energy from interacting with parallel A/IC waves, and parallel A/IC waves are neither damped nor amplified [8, 13]. The shape of the contour $\eta = 0.075v_\mathrm{A}^2$ is shown in the top panel of Figure 2.

When protons interact with oblique A/IC waves with a single nonzero value of θ, they also undergo wave pitch-angle scattering along a set of nested, closed contours

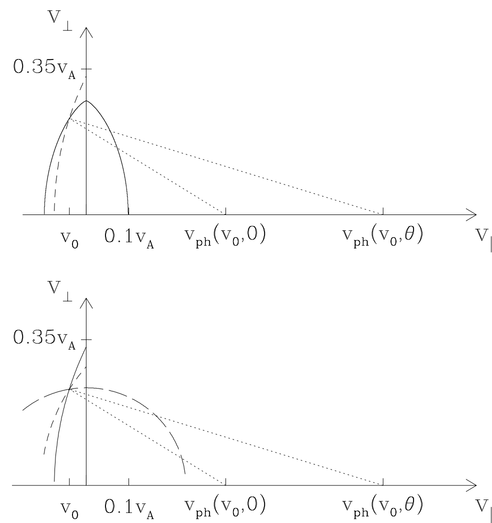

FIGURE 2. *Upper panel:* The proton distribution is taken to be constant along the $\eta =$ constant scattering contours for waves with $\theta = 0$, and to decrease as one moves to contours that are farther from the origin. The solid line is the $\eta = 0.075v_\mathrm{A}^2$ contour. Protons interacting with oblique waves will diffuse upward along the dashed line, gaining energy, and damping the oblique waves. *Bottom panel:* The proton distribution function is now taken to be constant along the scattering contours for oblique waves with some nonzero θ, and to decrease as one moves to contours that are farther from the origin. The solid line illustrates one such contour. Protons interacting with waves with $\theta = 0$ will diffuse down the density gradient along the short-dashed line, losing energy, and amplifying the $\theta = 0$ waves. The long-dashed line is a contour of constant energy in the plasma frame.

in the $v_\parallel - v_\perp$ plane. However, at a fixed $v_\parallel \ll v_\mathrm{A}$, the parallel phase velocity $\omega/k_\parallel$ of resonant oblique waves, denoted $v_\mathrm{ph}(v_\parallel, \theta)$, is greater than the parallel phase velocity of resonant parallel waves, $v_\mathrm{ph}(v_\parallel)$ [12]. Thus, at a fixed point in the $v_\parallel - v_\perp$ plane, the oblique-wave scattering contour has a larger slope than the parallel-wave scattering contour, as illustrated in Figure 2, which is adapted from [12]. In the top panel of Figure 2, we take the proton distribution function to be constant along the $\eta =$ constant scattering contours of $\theta = 0$ waves, and the solid-line curve corresponds to $\eta = 0.075v_\mathrm{A}^2$. Protons at $v_\parallel = v_0$ that are scattered by oblique A/IC waves with some nonzero value of θ will scatter along the dashed-line trajectory, which corresponds to constant energy as measured in a reference frame moving at velocity $v_\mathrm{ph}(v_0, \theta)$ along the magnetic field. [We have artificially increased $v_\mathrm{ph}(v_0, \theta)$ relative to $v_\mathrm{ph}(v_0, 0)$ in both panels of Figure 2 to make the figure easier to read.] If we take f to be a decreasing function of η, then there will be a net diffusive flux of protons upward along this dashed line, resulting in an increase in particle energy and damping of oblique waves. Elsewhere, we have calculated analyt-

ically the damping rate of oblique A/IC waves assuming that $f = f(\eta)$ [12].

In the bottom panel of Figure 2, we take the proton distribution to be constant along the scattering contours corresponding to waves with a single nonzero value of θ. One of these contours is now drawn with a solid line. Protons at $v_\parallel = v_0$ interacting with A/IC waves with $\theta = 0$ will scatter along the short-dashed line in this panel, which locally corresponds to an $\eta =$ constant curve. If we take the proton distribution to decrease as one moves to closed (solid-line) contours that are farther from the origin, then there will be a net diffusive flux of protons downward along the short-dashed line in the bottom panel of Figure 2. In this case, the protons will lose energy, and waves with $\theta = 0$ will be amplified.

Based on these arguments, we make the following conjecture. If some mechanism generates high-frequency A/IC waves with a range of θ values, and if the form and evolution of the proton distribution function are dominated by wave-particle interactions, then interactions involving waves with nonzero θ will act to make the constant-f contours steeper in the $v_\parallel - v_\perp$ plane than the $\eta =$ constant scattering contours of the parallel waves. This in turn will lead to the amplification of waves with $\theta = 0$ and cause the angular distribution of the waves at large k_n that resonate with the protons to become sharply peaked around $\theta = 0$. Wave-particle interactions will then become dominated by waves with $\theta = 0$, f will become approximately constant along surfaces of constant η, and oblique waves will be damped. In the next section, we describe numerical calculations that support this conjecture.

NUMERICAL CALCULATIONS

In the quasilinear theory of resonant wave-particle interactions, protons diffuse in velocity space as described by the equation

$$\frac{\partial f}{\partial t} = \lim_{V\to\infty} \frac{\pi q^2}{4m_p^2} \sum_{n=-\infty}^{\infty} \int d^3\vec{k} \frac{(2\pi)^{-3}}{V} \frac{1}{v_\perp} G v_\perp \delta(\omega_{kr} - k_\parallel v_\parallel - n\Omega_p) |\psi_{n,k}|^2 G f, \qquad (6)$$

where $G = \left(1 - k_\parallel v_\parallel/\omega_{kr}\right)\partial/\partial v_\perp + (k_\parallel v_\perp/\omega_{kr})\partial/\partial v_\parallel$, $\psi_{n,k} = E_k^+ J_{n+1}(k_\perp v_\perp/\Omega_p) + E_k^- J_{n-1}(k_\perp v_\perp/\Omega_p)$ (where we have set $E_{k,z} = 0$), V is the volume, J_n is the Bessel function of order n, $E_k^\pm = E_{kx} \pm iE_{ky}$, and $\mathbf{E}_k$ ($\mathbf{B}_k$) is the Fourier transform of the electric (magnetic) field [8, 9]. To integrate equation (6) numerically, we discretize velocity space using cylindrical coordinates with 100 grid cells spanning the interval $0 < v_\perp < 0.5\, v_\mathrm{A}$ and 100 grid cells for the interval $-0.5\, v_\mathrm{A} < v_\parallel < 0$. We discretize k-space using spherical coordinates with 30 grid cells spanning the interval $0 < k < 400\ \Omega_p/v_\mathrm{A}$ and 10 grid cells for the interval $0 < \theta < \pi/2$. We assume cylindrical and reflectional symmetry.

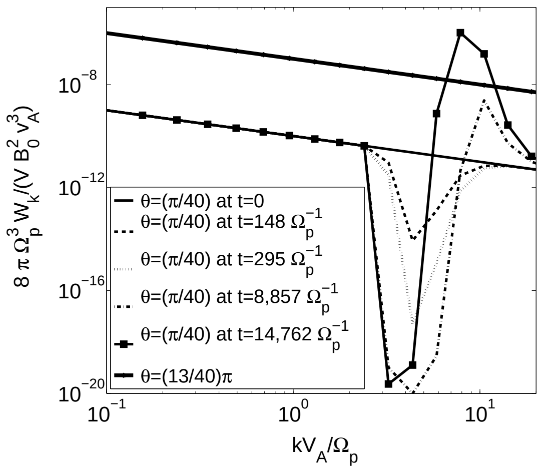

FIGURE 3. The power spectrum of the smallest-θ waves are shown at different times, and are compared to the fixed power spectrum of the waves with $\theta = 13\pi/40$.

The wave energy per unit volume in k space is $W_k = \left\{\mathbf{B}_k^* \cdot \mathbf{B}_k + \mathbf{E}_k^* \cdot [\partial(\omega \underline{\underline{\varepsilon}}_h)/\partial\omega] \cdot \mathbf{E}_k\right\}/(8\pi)$, where $\underline{\underline{\varepsilon}}_h$ is the hermitian part of the dielectric tensor [9]. We evolve W_k in time using "detailed energy conservation," i.e., by keeping track of the change in the particle energy $\Delta\mathscr{E}$ resulting from waves in each wavenumber bin, and deducting $\Delta\mathscr{E}$ from the wave energy in that wavenumber bin. It can be shown that this method is equivalent to evolving the waves using the analytic formula for the damping or growth rate γ_k given by [14] for the limit in which $|\gamma_k| \ll |\omega_{kr}|$.

We integrate equation (6) for f (and the corresponding equation for W_k resulting from detailed energy conservation) using an implicit time stepping algortihm, the biconjugate gradient stabilized method [15]. We hold the wave power spectrum W_k fixed at the value $10^{-7}k^{-1}VB_0^2v_\mathrm{A}^2/(8\pi\Omega_p^2)$ for $\theta = (13/40)\pi$. At all other values of θ, we initially set $W_k = 10^{-10}k^{-1}VB_0^2v_\mathrm{A}^2/(8\pi\Omega_p^2)$ and then we allow W_k to vary in time. The proton distribution function f is initially Maxwellian with a thermal speed $\sqrt{<v^2>}$ of $0.012\ v_\mathrm{A}$. The minimum value of θ at the cell center in the run that we present is $\theta_\mathrm{min} = \pi/40$. We note that the scattering contours for waves with $\theta = \theta_\mathrm{min}$ are very similar to the $\eta =$ constant lines.

At early times, all the waves are damped by interacting with thermal particles as shown in Figure 3. Waves with $\theta = (13/40)\pi$, which are initially dominant, cause particles to diffuse along the relatively steep scattering contours shown by the dotted lines in Figure 4. When the

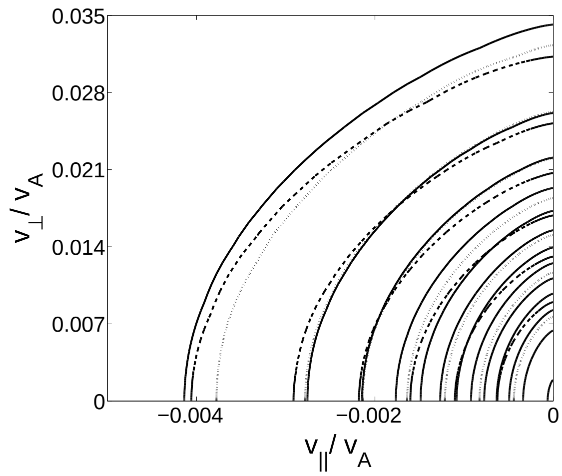

FIGURE 4. At $t = 8,857\Omega_p^{-1}$, the contours of constant f (solid lines) become steeper than the scattering contours of the waves with $\theta = \pi/40$ (dashed lines) and almost aligned with the scattering contours of the waves with $\theta = (13/40)\pi$ (dotted lines). Waves with $\theta = \pi/40$ become unstable and subsequently grow in amplitude.

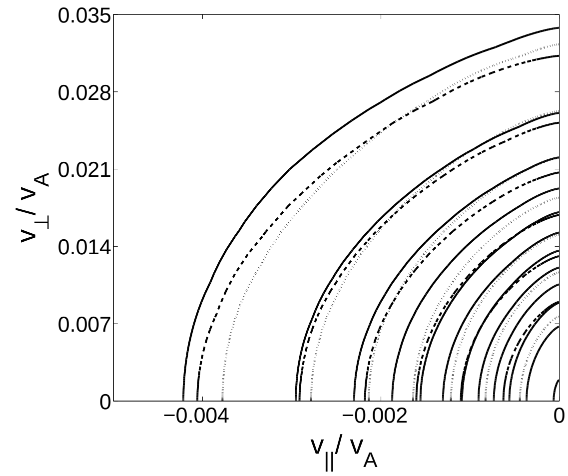

FIGURE 5. At $t = 14,762\Omega_p^{-1}$, the amplitudes of the waves with $\theta = \pi/40$ exceed the fixed amplitudes of the waves at $\theta = (13/40)\pi$ and the contours of constant f (solid lines) become almost aligned with the scattering contours of the waves with $\theta = \pi/40$ (dashed lines).

contours of constant f in the simulation become steeper than the scattering contours of the waves with $\theta = \theta_{\rm min}$, waves with $\theta = \theta_{\rm min}$ are amplified. After the energy in waves with $\theta = \theta_{\rm min}$ exceeds the energy in waves with $\theta = (13/40)\pi$, the small-θ waves begin to dominate, and the contours of constant f start to align with the scattering contours of the waves with $\theta = \theta_{\rm min}$, as shown in Figure 5. Even though we ongoingly input energy only into oblique waves, (quasi) parallel waves at $\theta = \theta_{\rm min}$ ultimately dominate, and the distribution function evolves to a state in which $f \simeq f(\eta)$

DISCUSSION

Isenberg [16] has shown that A/IC waves with $\theta = 0$ are unable to explain the heating and acceleration of protons in the fast solar wind, primarily because f relaxes towards a state in which $f \simeq f(\eta)$, after which the protons are only weakly heated by the waves. In contrast, we have shown that when protons are heated by oblique A/IC waves, the distribution function does not relax towards a state in which oblique-wave damping vanishes. Instead, f again approaches a state in which $f = f(\eta)$, and oblique waves continue to damp on the protons, causing protons to diffuse across $\eta =$ constant surfaces in the $v_\perp - v_\|$ plane. Because of this, oblique A/IC waves have the potential to be more effective than $\theta = 0$ waves at heating protons in the corona and solar wind.

ACKNOWLEDGMENTS

This work was supported by NSF Grants ATM-0851005 and 0850205, by DOE under Grant DE-FG02-07-ER46372, by NSF-DOE Grant AST-0613622, and by NASA under Grants NNX07AP65G and NNX08AH52G.

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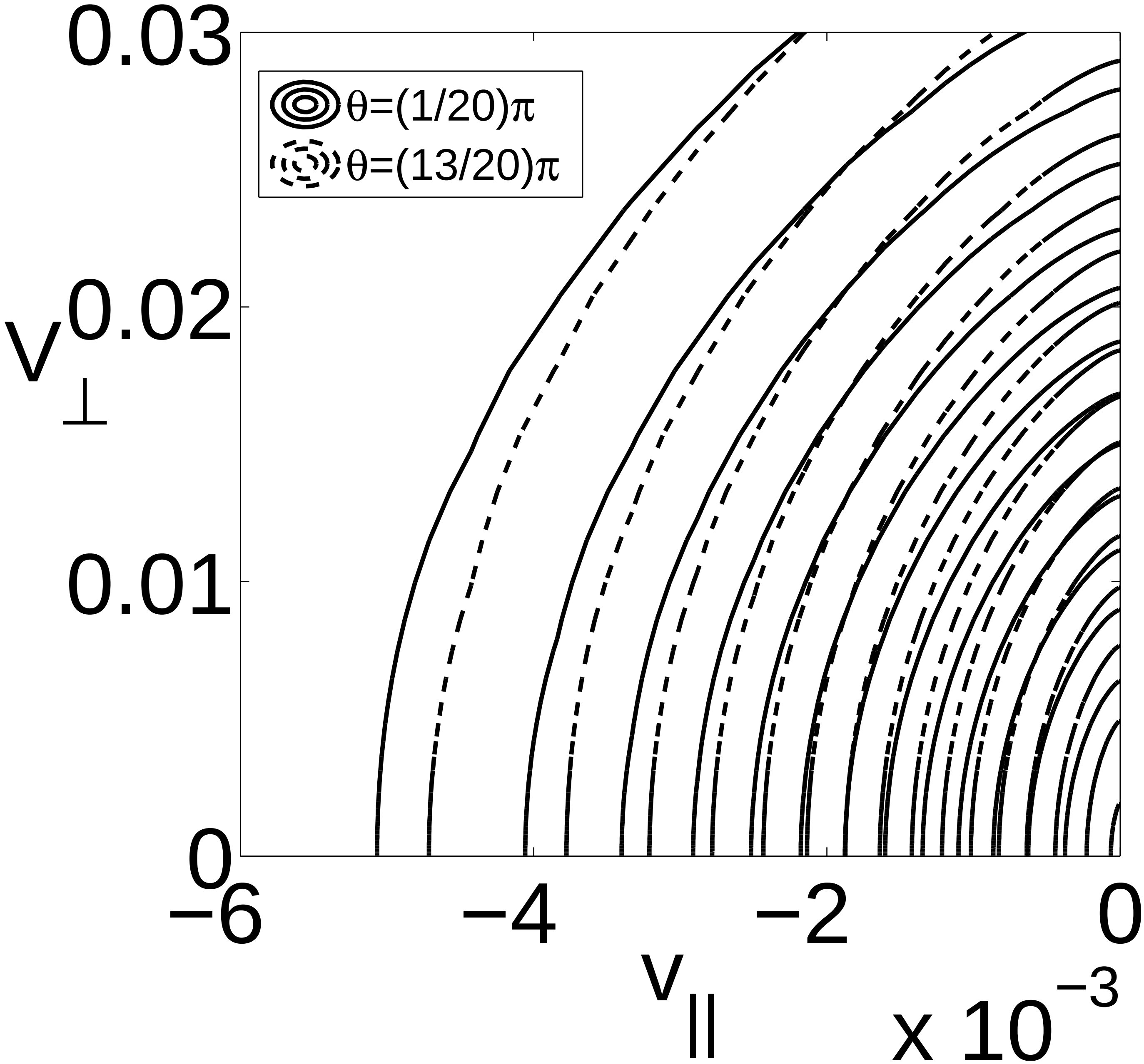

θ=(1/20)π
θ=(13/20)π
0.03
0.02
0.01
0
$v_\perp$
−6
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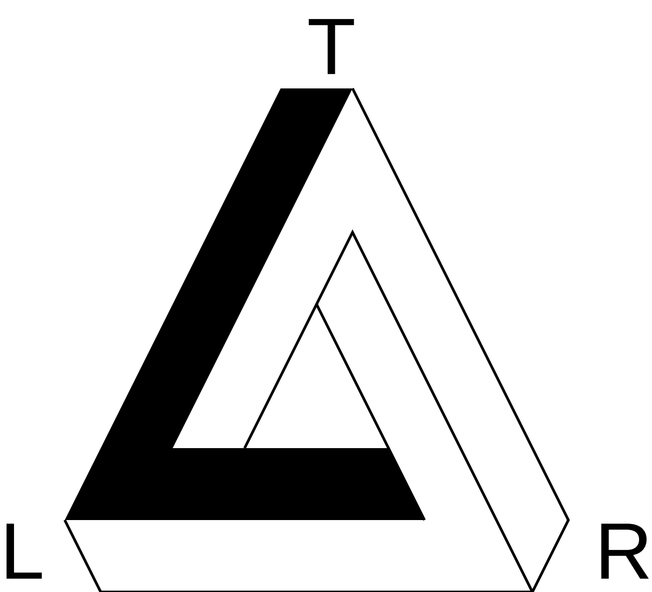
x 10^{-3}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